The article analyzes all stages of production of both raw materials and finished agricultural products common for the modern agricultural production in Ukraine. Both the external and internal factors influencing the primary and secondary production of agricultural products were studied. The markets for agricultural raw materials and finished products, namely, the national and international markets for agricultural products, were also considered.

Based on the results of analysis, the economic-mathematical model of production and processing of agricultural products as a single economic system has been constructed. The model was built taking into account the most vital requirements of each of the selected stages. Each of the five stages of production, that were distinguished, was described using mathematical formulas. For some of the steps, the Cobb-Douglas multiplicative production function, modernized for the specifics of agricultural production, was used. The function used includes the influence of the areal factor on the gross agricultural production, as well as classic labor and capital factors. The problem of increasing the efficiency of the agricultural sector of Ukraine's economy was formulated and solved.

Using this economic-mathematical model of production in the development of agricultural policy and macroeconomic planning will allow to consider changes in the economy of the agricultural and processing industry, to assess the possible consequences of new activities in state economic policy, implementation and monitoring of the long-term negative and positive trends of production and processing of agricultural products. Moreover, due to this model, it could be possible to increase the volume of earnings in foreign currency, which, in turn, can be used as investment not only in agricultural but also in other sectors of the national economy.

Keywords: agriculture, agricultural production, mathematical modeling, economy of Ukraine.

У статті проаналізовано усі етапи виробництва як сировини, так і готової продукції сільського господарства, що є характерними для сучасного сільськогосподарського виробничого процесу України. Було розглянуто як зовнішні, так і внутрішні фактори, що впливають на первинне та вторинне виробництво сільськогосподарської продукції. Також було розглянуто ринки збуту сільськогосподарської
сировини та готової продукції, а саме національний та міжнародний ринок товарів сільського господарства.

За результатами аналізу побудовано економіко-математичну модель виробництва та переробки сільськогосподарської продукції як єдиної економічної системи, з урахуванням вимог кожного з виділених етапів. Кожен з 5 виділених етапів виробництва був описаний за допомогою математичних формул. Для деяких з етапів було використано мультиплікативну виробничу функцію Кобба-Дугласа, модернізовану під специфіку сільськогосподарського виробництва. Функція, що була використана, враховує вплив земельного фактора на валове сільськогосподарське виробництво, наряду з класичними факторами праці та капіталу. Була поставлена та вирішена проблема підвищення ефективності сільськогосподарського сектора економіки України.

Використання цієї економіко-математичної моделі виробництва при розробці сільськогосподарської політики та макроекономічному плануванні надасть змогу розглянути зміни в економіці сільськогосподарської та переробної промисловості, оцінити можливі наслідки нових видів діяльності у державній економічній політиці, вивчити необхідний ступінь свободи їх реалізації та провести моніторинг довгострокових негативних та позитивних тенденцій у виробництві та переробці сільськогосподарської продукції. Також, завдяки цій моделі, можна збільшити обсяги валютних надходжень, які, у свою чергу, можуть використовуватися як інвестиції не тільки у сільськогосподарську, а і в інші галузі національної економіки.

Ключові слова: сільське господарство, сільськогосподарське виробництво, математичне моделювання, економіка України.

**Introduction.** Ukraine has a significant potential for the development of the agrarian sector. Rural territories make up about 90% of its territory, a third of the population lives here. In addition, Ukraine has a convenient geographical location, mild climate and rich resource potential.

For Ukraine the agricultural sector has always been one of the prioritized areas of both internal and external economic development. Nowadays, agriculture forms 12% of GDP, and the agrarian sector itself has become the largest source of currency that comes to the state, which provides the power of the Ukrainian hryvnia [1]. With the reorientation of Ukraine's economic development into the EU, the agricultural sector has gained new possible markets that need to be saturated. However, for this purpose it is necessary to improve the management systems of agricultural enterprises at the state level.

This research topic is very important due to the need to solve numerous problems in the field of production management of numerous national agro-industrial enterprises.

There are numerous works devoted to the problems of economic-mathematical modeling in the production management. Such scientists as S. Volkov, S. Tsarev, A. Varlamov, K. Kiryukhina, A. Kupchinenko, L. Tverdovskaya and others have described the systems of macroeconomic models of planning and analysis of the development of the agro-industrial sector on the national and regional levels.

**Setting objectives.** The aim of the research is to construct an economic-mathematical model, which describes all the processes that take place during production at the enterprises of agroindustrial complex at macro level and to
develop organizational and economic principles and constructive proposals regarding the formation of effective production management system of agricultural enterprises.

The aim determines the following tasks:
- to analyze the current state, trends and problems of the agroindustrial complex;
- to mathematically describe processes of agricultural production and unite them into a single model;
- to optimize the activities of the industry in the light of current trends and needs in the national and world markets;
- to develop scientific and practical recommendations for implementation of effective methods of managing the production activities of enterprises at different levels.

**Methodology.** The main method of research is the method of modeling economic processes in agriculture, which determines the complex of various techniques that have become widespread in science and practice in recent years. Theoretical basis of the study is accumulated scientific experience in the works of domestic and foreign scientists.

The information basis for the study included legal and normative documents, materials and reports of state services and scientific institutions, special literature, as well as the results of monographic and sociological studies of other researchers.

**Research results.** The main production processes are being modeled, ie, direct agricultural production and food production without the production of feed, equipment, machinery, construction, infrastructure, etc. The model is based on the dividing of economic and technological processes happening in agriculture into 5 stages, according to the specifics of agribusiness:
2. Primary production in agriculture.
5. Sale of agricultural goods.

Despite the fact that the authors of the original model singled out only 4 stages, we added another one in order to adapt the model to the realities of agricultural production in Ukraine as much as possible [8].

It should be noted, that the sale of agricultural raw materials and finished products industry at stages (3) and (5) is carried out both on the national and international markets. In general, the process of production and distribution of agricultural products is depicted in Figure 1.
The first two stages are the stages of primary agricultural production. Distribution of the agricultural raw materials for their further processing is allocated to the third stage. Secondary agricultural production is the fourth stage. The final stage is the sale of finished products of the food industry.

The first stage "Goods processes in agriculture" describes a completely certain set of economic, organizational and technological measures in agricultural production for obtaining and forming an intermediate product. This complex allows to highlight the first production cycle. After that, the intermediate product is directed and completely consumed in the primary production.

The separation of this stage happened due to the existing peculiarities of agricultural production associated with the length of the production cycle. In order to describe the gross output of goods processed in agriculture, the authors of the model proposed a two-factor Cobb-Douglas production function.

\[ X'' = F(C'', L'') \] (1)

However, in 2014, the article authored by A. Skrypnyk and T. Yara was published, where the economic indicators of the agrarian sector of Ukraine from 1998 to 2012 were analyzed [2]. The authors compared three variants of the production function for the agricultural sector of the economy and found out which one of the suggested variations could determine the relationship between the production factors most accurately.

It should be noted that when using a standard model (input parameters: labor and capital), the labor parameter has a negative coefficient of elasticity. This means a 1% reduction in labor force leads to a similar increase in output. Moreover, it is impossible to reject the hypothesis regarding the values of labor and capital elasticity based on the assessment of the significance level.

The second model (input parameters: labor, capital, area) slightly increases the determination coefficient, leaving insignificant coefficients of elasticity versus capital and area.
Therefore the standard models with two (labor, capital) or three (labor, capital, area) production factors can hardly be used to analyze and forecast the contribution of certain factors to agrarian production.

However, the last model includes the time factor and has the most significant indicators of adequacy (determination coefficient), where all the elasticity indicators are positive and have a significance level that does not exceed 2.5%.

Thus, the last model describes the first stage of agricultural production the most accurately:

\[ X_t'' = a_0 e^{a_1 C_t^{\alpha_1}} L_t^{\alpha_2} S_t^{\alpha_3} \] (2)

where:

- \( X_t'' \) – intermediate product of goods processed by agricultural products
- \( C_t \) – capital in the form of fixed and floating assets
- \( L_t \) – labor costs for the production of an intermediate product
- \( a_0 \) – coefficient of neutral technical progress
- \( a_1 \) – autonomous growth rate
- \( \alpha_1 \) – coefficient of intermediate product elasticity versus capital
- \( \alpha_2 \) – coefficient of intermediate product elasticity versus labor
- \( \alpha_3 \) – coefficient of intermediate product elasticity versus area

As can be noted, in addition to the traditional factors of production (capital \(- C\) and labor \(- L\)), the area \( S \) and the autonomous growth factor \(- a_0 \) were added. The last parameter takes into account the impact of scientific and technological progress (increase in the efficiency of the use of capital) and reflects the growth rate of production.

In the second stage "Primary production in agriculture" an intermediate product, created in the first stage, is consumed for the production of the final agricultural product. The consumption and change of the intermediate product affects a number of factors (or resources) such as labor, feed, fertilizers, irrigation and equipment.

Thus, the following regression equation is used to describe the production processes in "Primary production in agriculture":

\[ X_t' = a_0 + a_1 (1 - \omega)X_t'' + a_2 L_t' + a_3 H_t + a_4 O_t + a_5 t \] (3)

where:

- \( X_t' \) – the final product of the agriculture
- \( L_t' \) – labor costs for the production of finished products
- \( H_t \) – cost of fertilizers
- \( O_t \) – cost of equipment
- \( \omega \) – part of the yield loss
- \( t \) – time variable
- \( a_0, a_1, ..., a_5 \) – parameters of the equation

The third stage "Sale of agricultural raw materials" describes the acquisition by the processing enterprises of products obtained in the second stage. However, for our country, the sale of agricultural raw materials is usually happening not only on
domestic but also on foreign markets. This means that part of the harvest will be purchased by domestic processing enterprises at national currency prices, while the rest of the final product will be exported and sold at prices in foreign currency.

For the mathematical description of the processes of the third stage you can use the following equations:

\[ X_{nat} = \beta (1 - \omega)(1 - \mu)X_t' * P_{nat} \]  
\[ X_{exp} = (1 - \beta)(1 - \omega)(1 - \mu)X_t' * P_{exp} \]

where:

\( X_{nat} \) – revenue from sales of finished products on the national market;
\( X_{exp} \) – revenue from sales of finished products on the international market;
\( P_{nat} \) – price of agricultural products on the national market;
\( P_{exp} \) – price for agricultural products on the international market;
\( \beta \) – part of the final agricultural product used for national production.
\( \omega \) – part of the production losses associated with the spoilage;
\( \mu \) – part of the final product used as the atrial for the subsequent part of the final product used to recover the liquidation and restoration of the industrial property in the first stage "Processing of goods in agriculture", ie for the formation of seeds, the formation of stocks etc.

Production processes of the fourth stage "Secondary production in agriculture", as in the first stage, are most accurately described by the production function. The production depends on the size and combination of resources; hence, there is a direct dependence between the result of production and resource stocks. Therefore, everything related to the production function in the first stage is true for the production function used in the fourth stage. Under this assumption of the same conditions and rules, the following function of production was chosen.

\[ Y_t = a_0C_t^{\alpha_1}L_t^{\alpha_2}(X_t + \Delta X)^{\alpha_3} \]

where:

\( Y_t \) – final product of food industry;
\( C_t \) - capital in the form of fixed and floating assets;
\( L_t \) – labor costs for the production of food products;
\( \Delta X \) – acquisition of additional raw materials and materials;
\( a_0 \) – coefficient of neutral technical progress;
\( \alpha_1, \alpha_2, \alpha_3 \) – coefficients of elasticity.

The production function (5), in comparison with the first stage of the function (2) for the production of the processing industry (enterprise) uses three resources: capital \( C \), labor force \( L \) and raw material \( X \). The final product of agriculture \( X \) serves as a resource for the fourth stage of production.

The fifth stage is the final stage of production and technological processes of agricultural production. It is characterized by the distribution of the finished product between the internal and external markets and is described by the following equations

\[ Y_{nat} = \gamma * Y_t * P_{nat} \]
\[ Y_{exp} = (1 - \gamma) * Y_t * P_{exp} \] (6.2)

where:
\( Y_t \) – quantity of produced products;
\( Y_{nat} \) – revenue from sales of finished products in the national market;
\( Y_{exp} \) – revenue from sales of final products in the international market;
\( P_{nat} \) – price of agricultural products on the national market;
\( P_{exp} \) – price for agricultural products in the international market.
\( \gamma \) – part of final industrial output sold in the national market.

**Figure 2 - Mathematical description of processes in agricultural production**

This mathematical model of the production and processing of agricultural products as a single economic system will allow us to consider changes in the economy of the agricultural and processing industry, to evaluate the possible consequences of new activities in the state economic policy, to study the necessary degree of freedom of their implementation and to monitor the long-term negative and positive trends in production and processing of agricultural products.
Now this model should be specified for our task. All major agricultural products cultivated in Ukraine can be divided into the following 8 groups:

1. grains and legumes;
2. sugar beet;
3. sunflower;
4. rapeseed;
5. soy;
6. potato;
7. vegetables;
8. fruit.

Let us introduce the variable $S^{(i)}$ – the area of land in thousands of hectares, which accordingly occupies each of the cultures in the first stage.

The goal of the task is to maximize profits from the export of both agricultural raw materials and finished products of the food industry, taking into account all stages of the production process. That is why the target function has the following form:

$$Z = X_{exp} + Y_{exp} \rightarrow \max$$  (7)

The task has the following restrictions

1. The area occupied by each culture must be inalienable

   $$S^{(i)} \geq 0, i \in (1, 8)$$  (8)

2. The amount of area occupied by crops should not be more than the total area of arable land.

   $$\sum_{i=1}^{8} S^{(i)} \leq 29.4$$  (9)

3. Before selling agricultural raw materials, it is necessary to satisfy, first of all, the domestic demand for each of the crops

   $$\beta\left(X''_{t} \cdot S^{(i)}_{t}\right) \geq N^{(i)}_{t}, i \in (1, 8)$$  (10)

   $N^{(i)}_{t}$ – domestic demand for agricultural raw materials of the second grade

The practical implementation of the model showed the following quantitative and qualitative structures of the areas occupied by agricultural crops and the volume of production of agricultural raw materials and finished products
Figure 3 - The calculated area occupied by each culture

Figure 4 - Calculated production of raw agricultural products

Figure 5 - Calculated production of finished agricultural products
**Conclusions.** The result of the work is theoretical generalization and a new approach to the solution of the problem of modeling the Ukrainian agricultural market.

As a result, the economic-mathematical model of Ukrainian agribusiness enterprises at the macro level was built. This model shows the great practical possibilities of modeling the production and processing of agricultural products as a single economic system. Moreover, it makes it possible to assess the effects of new activities in the state economic policy and improve the economic performance of enterprises in the agroindustrial complex in general.

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