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## MODELING AN EFFECTIVE STRATEGY OF ADVERTISING INVESTMENTS OF FIRM IN THE COMPETITIVE MARKET

## МОДЕЛЮВАННЯ ЕФЕКТИВНОЇ СТРАТЕГІЇ РЕКЛАМНИХ ВКЛАДЕНЬ ФІРМИ НА КОНКУРЕНТНОМУ РИНКУ

In this article, a dynamic model of the company's sales dependence on advertising costs is constructed. In a competitive market, advertising is not only a means of increasing demand, but also a competition tool at a level with the price. Advertising allows you to increase sales, but the dependence of increasing demand from increased advertising costs is not linear. If you increase your advertising costs after a certain value, their efficiency will decrease. This model takes into account the dependence of advertising efficiency on the change in the market share of advertising costs of the firm. It is shown that the complexity of capturing a new audience will grow geometrically, and therefore the efficiency of advertising will decrease geometrically. Together with the saturation of advertising, this is reflected in the parameter  $i_b$ . Advertising does not work instantly, since its inception, the information is accumulated in the minds of people with each meeting with advertising and reaches the peak after a certain time. If the ads become smaller, then people will forget it more quickly than they will be reminded of it, and the peak of their knowledge of the product will fall. When ads appear to people, it takes some time until their knowledge of the product reaches a peak above which they will not be able to assimilate advertising, because they will forget its details at the same speed. As a rule, this requires about two weeks of advertising. The second manifestation of the delay is related to knowledge of the product as a whole. This requires a relatively long period of time. The model reflects the effect of delayed ad exposure, which is expressed in the so-called "inertial sales". They can form the bulk of the sales, due to advertising. That increases the importance of promotional investments in the long run, so that real-time advertising may not pay off itself, but eventually turns into profit. In the model, the delay associated with knowledge of the product as a whole is taken on average about a year. So it takes into account seasonality of sales volumes. And given the dynamics of market change over the past period, their corresponding change is taken into account. In the model, "inertial sales" are calculated in proportion to the parts from the required time, where changes take place. General sales consist of "inertial sales" and those are caused by short-term advertising influence.

**Keywords:** the share of the advertising market, the effectiveness of advertising, the dynamics of market change, delay in the influence of advertising.

У статті побудована динамічна модель залежності продажів фірми від витрат на рекламу. На конкурентному ринку реклама  $\epsilon$  не тільки засобом підвищення попиту, але й інструментом конкурентної боротьби нарівні з ціною. Реклама дозволяє збільшити обсяг продажів, однак залежність підвищення попиту від підвищення рекламних витрат не  $\epsilon$  лінійною. Після певного рівня рекламних вкладень їх ефективність буде спадати. В моделі

враховується залежність ефективності реклами від зміни частки ринку рекламних витрат фірми. Показується, що складність захвату нової аудиторії буде зростати геометрично, тому й ефективність реклами буде спадати геометрично. Разом із насиченням рекламою це відображається у параметрі  $i_{\rm b}$ . Реклама не діє миттєво, з моменту її початку інформація накопичується у свідомості людей з кожною зустріччю з рекламою і досягає пику через певний час. Якщо реклами стане менше, то люди почнуть забувати її швидше, ніж їм будуть нагадувати про неї, і пик їх знань про товар впаде. При появі реклами людям потрібен деякий час, поки їх знання про товар не достигне пику, вище якого вони не зможуть засвоювати рекламу, через те, що будуть забувати її деталі з рівною швидкістю. Як правило на це потрібно близько двох тижнів реклами. Другий прояв затримки впливу реклами пов'язаний із знанням про товар у цілому. Для цього потрібен порівняно великий період часу. В моделі відображена властивість впливу реклами із затримкою, що виражається в так званих "інерційних продажах". Вони можуть сформувати основну частину продаж, обумовлених рекламою. Що підвищує значимість рекламних вкладень в довгостроковому періоді впродовж до того, що реклама в поточному періоді може не окупати себе, але з часом переходить в прибуток. Прояв затримки, пов'язаний із знанням про товар у цілому, в моделі береться в середньому біля року. Таким чином врахована сезонність обсягів продажів. А з урахуванням динаміки зміни ринку за минулий період ураховується їх відповідна зміна. В моделі "інерційні продажі" розраховуються пропорційно частині від необхідного часу, де відбуваються зміни. Загальні "інерційних продажів" продажі складаються ma тих. обумовлені короткотерміновим впливом реклами.

**Ключові слова:** частка рекламного ринку, ефективність реклами, динаміка зміни ринку, затримка впливу реклами.

**Introduction.** Mathematical formalization of such complex and multifaceted activities as marketing is hardly possible. Nevertheless, for individual marketing elements, mathematical modeling is entirely acceptable and can be useful. It is primarily about the costs of advertising. Determining the best advertising costs is one of the complex challenges faced by the leadership of any firm.

In a competitive market, advertising is not only a means of increasing demand, but also a competition tool along with price. In addition, the effect of advertising is delayed. [1]

Advertising allows you to increase sales, but the dependence of increasing demand from increased advertising costs is not linear. If you increase your advertising costs after a certain value, their efficiency will decrease.

Since there are other companies selling similar products or services on the market and your firm shares the same audience with them, one part of people will buy your product, and the other part is the product of competitors. They will make their choice based on many criteria, including advertising.

In certain periods your product will be bought more. From the point of view of advertising, this means that the number of people who can be promoted by advertising will buy more, and hence advertising costs should be higher.

Advertising does not work instantly, since its inception, the information is accumulated in the minds of people with each meeting with advertising and reaches the peak after a certain time. If the ads become smaller, then people will forget it more quickly than they will be reminded of it, and the peak of their knowledge of the product will fall. If there is no advertising at all, their knowledge will be extinguished for some time until they reach the minimum level. [2]

A simple dynamic model of controlling advertising costs was formulated in [3] and has the form:

$$\int_0^T x(t) dt \xrightarrow{A(t)} max$$

under conditions  $\dot{x} = -a \cdot x + k \cdot A \left[ 1 - \frac{x}{M} \right], x(0) = x^0, 0 \le A(t) \le \bar{A},$ 

where x(t) is the sales volume at time point t, A(t) is the level of advertising activity of the enterprise, M is the market volume, T, a, k,  $\bar{A}$  are given positive parameters.

The first term in the differential equation is explained by the limited volume of output potentially destined for realization in a certain period of time. The second term describes the possible reaction of the product realization process depending on the advertising activity in the unoccupied part of the market.

On the basis of the model under consideration, in [4] the existence of the minimum admissible level of expenditures for advertising activity is proved, upon attainment of which there is a definite effect on the realization. In addition, the author [4] considers the generalization of the initial model, introducing a fixed lag of the delayed influence of advertising activities on sales as follows:

$$x(t) = -a \cdot x(t) + k \cdot A(t-\tau) \left[ 1 - \frac{x(t)}{M} \right]$$

and also  $A(\xi)=A_n$  in the case  $\xi\in[-\tau,0]$ .

For the given case it is proved that the optimal expenses for advertising and the asymptotic level of sales will remain the same as in the solution of the model without taking into account the delayed influence of advertising [4].

Another generalization of the initial model in [4] consists in taking into account the actions of competitors in the market of a certain product. In this case

$$\dot{x_0} = -a \cdot x_0 + k \cdot A(t) \left[ 1 - \frac{\left(1 + d_{k/d_0}\right) \cdot x_0}{M} \right],$$

where  $d_k$  is the cumulative share of competitors in the market,  $d_0$  is the share of the enterprise in question on the market,  $x_0(t)$  is the sales volume of the enterprise under consideration at time t.

One can see that in the models considered, seasonality of sales did not appear. The volume of products, potentially intended for sale in a certain period of time, is limited. The capacity of the market, the market share of the enterprise under

consideration and the cumulative share of its competitors are unchanged. The possibility of capturing competitors' clients is not reflected. Short-term and long-term effects of advertising on consumers are also not considered.

**Setting objectives.** Consider the economic decision-making strategy to determine the cost of advertising in a competitive market, taking into account seasonality and the effects of advertising on consumers: saturation and delays.

We will construct a dynamic model of the firm's sales at the time *t*, based on the sales data of the firm, the share of its advertising market in the present and previous periods.

**Methodology.** In analyzing the influence of advertising and building a model, a dynamic approach and models with a time delay were used.

**Research results.** Advertising costs are denoted by *A*. In this case, advertising has some efficiency, which is quite unstable. It is influenced by two main sources – internal and external.

Internal influence shows how the investment in advertising is converted into sales. For example, where were the money invested – in television, the Internet, the radio, or elsewhere. External factors of influence – this is something we cannot influence. Mostly it's our customers and competitors.

In this case, when modeling the efficiency of advertising on the basis of it is necessary to take averaged data for past periods.

This means that we have some correlation between the money that we have additionally invested in advertising and the increase in sales. This will be our advertising efficiency, which we will denote by b.

At the same time, the efficiency of advertising varies from its quantity, namely, our competitors and customers are sensitive to the amount of our advertising.

Competitors also have their own advertising, and in response to an increase in our advertising will increase their own, which will reduce our efficiency, because they operate on the same audience. Therefore, for convenience, advertising costs can also be recorded as  $\frac{A}{R}$  – the share of advertising market, which includes our competitors. Here R is an advertising market (the volume of advertising of our company and competitors (in money), which acts on customers).

In this case, the dependence will have the following form:

$$b=\chi\cdot\tfrac{A}{R},$$

where x is the saturation parameter by our advertising.

If we want to change the market share of advertising, we will come across a number of consequences. First of all, this change will affect the customers of our competitors, and some of them will become ours (Figure 1).

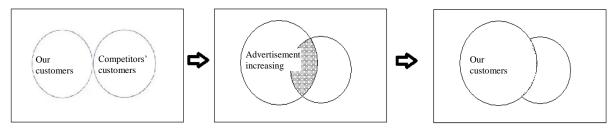


Figure 1 – Consequences of increasing the share of the advertising market

It will also affect our old customers and, accordingly, their saturation with advertising. Our expansion will cause the competitors to react accordingly. Even if there is no reaction, it will become more and more difficult to capture his audience, as there are numerous external factors that influence the choice of clients. There are territorial features, product features, independent recommendations and many others (Figure 2).

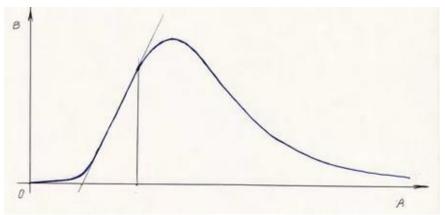


Figure 2 – Dependence of advertising efficiency on the size of advertising investments

Therefore, changing the efficiency of advertising relative to the change in the size of advertising costs will be as follows:

$$b(t,\tau) = \frac{{}_{A(t)}}{{}_{R(t)}}/i_b \frac{{}_{A(t)}/{}_{R(t)}/\frac{A(t-\tau)}{R(t-\tau)}},$$

where  $i_b$  is a certain positive number, which expresses the sum of all factors that respond to the advertising activities of the firm. The main ones are competitors and their clients.

The level of loyalty shows how stable the position of the clients of firms in choosing the product of the firm. When increasing of advertising investments of one firm, its pyramid of influence will grow and will capture a small part of the neighboring ones, which is the most bearish. Accordingly, in order to capture higher

shares, much more investments will be required. In a compilation with saturation of advertising, this is reflected in the parameter  $i_h$ .

Dependence  $i_b^{\frac{A(t)}{R(t)}/\frac{A(t-\tau)}{R(t-\tau)}}$  of the present on the previous period shows that the difficulty of capturing a new audience will increase geometrically, and therefore the efficiency of the advertisement will decrease, respectively, geometrically.

But this is the efficiency of advertising for the current period, and the impact of advertising has the ability to manifest itself with a delay.

For this reason, let's consider two main manifestations of the delay.

When ads appear to people, it takes some time until their knowledge of the product reaches a peak above which they will not be able to assimilate advertising, because they will forget its details at the same speed. As a rule, this requires about two weeks of advertising. [2]

The second manifestation of the delay is related to knowledge of the product as a whole. This requires a relatively long period of time. For different goods or services differently, but on average there is *about one year*. It depends on the amount of our advertising and the amount of advertisements competitors – we have all the same ratio  $\frac{A}{R}$ . It affects "inertial sales", which are further denoted by  $x_{(u)}$ .

They can form the bulk of the sales, due to advertising (depending on the product). That increases the importance of promotional investments in the long run, so that real-time advertising may not pay off itself, but eventually turns into profit.

The manifestation of the delay associated with knowledge of the product as a whole, we will assume an average of about one year. Thus, let's consider seasonality of sales volumes. And with the inclusion of the indicator of the dynamics of market change over the past period, we will consider their respective changes.

$$x_{(u)}\left(t,\tau\right)=x_{(u)}\left(t-1\right)*a(t)*\left(\frac{A(t)}{R(t)}/\frac{A(t-\tau)}{R(t-\tau)}\right)/i_{b}^{\frac{A(t)}{R(t)}/\frac{A(t-\tau)}{R(t-\tau)}}*\tau;$$

where a(t) is the dynamics of market change over the past period;  $\tau$  expresses a part where changes occur.

For example, we increased the market share of advertising over two months by 10%. So "inertial sales", taking into account all geometric penalties due to competitors and saturation, will change to a certain number, which we will divide by 6. (For the reason that we have a period of change of 1 year, and 2 months is 1/6 years.) That is, the effect of changing in the share of the advertising market is directly proportional to the part of the year for which these changes occurred.

In this dependence, the time lag is 1, which allows you to take into account the seasonality of sales based on sales of the past period, and the long-term effect of advertising. This, of course, is a certain simplification. The indicator of market dynamics reflects market trends.

General sales consist of "inertial sales" and sales, which are caused by short-term advertising influence. In general, our advertising costs will bring us the following sales:

$$x(t) = \sum_{\tau} x_{(u)}(t, \tau) + k * A(t) * b(t, \tau),$$

where k is a preset positive parameter.

Unlike [4, p. 71], unlimited production is taken into account in the presented research.

**Conclusions.** In the article, a dynamic model of the firm's sales dependence on advertising costs is constructed. The constructed model can be considered as a model with discrete time.

In this model, the seasonality of sales is manifested through dynamics. The volume of products, potentially intended for sale in a certain period of time, is unlimited. The volume of the advertising market, the share of the advertising market of the enterprise under consideration is not assumed to be constant. This affects the ability to capture customers' competitors.

It is shown that the complexity of capturing a new audience will grow geometrically, and therefore the efficiency of advertising will decrease geometrically. The model reflects the effect of delayed ad exposure, which is expressed in the so-called "inertial sales". They demonstrate the importance of advertising investment in the long term. General sales consist of "inertial sales" and those are caused by short-term advertising influence.

The above results can be used in calculating the volume of advertising investments in those or other periods, for example, to achieve maximum sales for a certain time, while limiting the total amount of advertising costs. Further research, for individual markets, are related to  $i_b$ , which expresses the sum of all reactions to advertising activities of the firm.

In the constructed model for acquaintance of buyers with the goods as a whole the period of time equal one year was supposed. If this period is not the same, the necessary changes for "inertial sales" must be made.

In the model, we can assume the dependence of "inertial sales" different from direct proportionality, on the time interval during which there are changes in the share of the advertising market.

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