PORTFOLIO MANAGEMENT WITH TIME SERIES ANALYSIS METHODS

The purpose of this research is to review and summarize the theoretical and methodological foundations of portfolio theory, methodological provisions for modeling an optimal portfolio using time series. To achieve the goal, the article used the modern portfolio theory of Harry Markowitz (MPT), which emphasizes the importance of diversification and quantitative assessment of risks and returns. The study also explores the adaptation of ARIMA and GARCH models for forecasting risk matrices, offering a nuanced approach to forecasting and utilizing market fluctuations to achieve optimal portfolio performance. The results of the study not only confirm the relevance of fundamental theories of portfolio management in the current environment, but also expand the toolkit for investors through the practical application of advanced time series analysis methodologies. The practical significance of the study is to provide investors with reliable strategies that will help them navigate and benefit from market dynamics, thereby maximizing investment results in the face of uncertainty inherent in financial markets.

Keywords: portfolio management, time series, covariance matrix, dynamic modeling, investment.

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The general aim to build an optimal portfolio of securities that will minimize risk, but maximize the investment results, using time series analysis approach.

Analysis of recent research and publications. In the rapidly evolving financial markets, characterized by constant fluctuations and significant uncertainties, portfolio management emerges as a critical discipline for optimizing investment returns while managing risk. The crux of modern portfolio management lies in leveraging forecasted risk matrices, a concept deeply rooted in Harry Markowitz’s pioneering Modern Portfolio Theory (MPT) from his 1952 dissertation “Portfolio Selection”. This theory underscores the importance of diversification and quantification of risks and returns through mathematical modeling. It posits that the optimal investment portfolio can be constructed by balancing the trade-off between the expected return and the associated risk, measured in terms of variance and covariance among assets. As financial markets continue to grow in complexity, the adaptation and application of sophisticated time series analysis methods, including ARIMA and GARCH models, play a pivotal role in forecasting and managing the dynamic risks, offering investors strategic tools to navigate and capitalize on market volatility for maximizing portfolio performance.

In the current conditions, where financial markets are subject to constant changes and significant fluctuations, it is important to have an effective mechanism for managing the equity portfolio to achieve the best possible outcomes. One of the key aspects of portfolio optimization is understanding the risks associated with investing in stocks. This study explores the use of forecasted risk matrices as a tool for building effective portfolio management strategies. Such an approach allows investors not only to adapt to changes in the financial environment but also to actively use these changes to maximize profits.

The foundation for creating an optimal stock portfolio is the concept developed by American scholar Harry Markowitz. He was one of the first to fully comprehend the benefits of forming an efficient equity portfolio in his 1952 dissertation “Portfolio Selection”. Modern Portfolio Theory (MPT) remains an important investment strategy, having a pivotal role in forecasting and managing the dynamic environment but also to actively use these changes to capitalize on market volatility for maximizing portfolio performance.

The target function that will be optimized:

\[ W = \frac{\alpha \cdot \sigma_p}{(1 - \alpha) \cdot R_p} \rightarrow \min. \]

Having defined the main criterion and constraints, we have an optimization problem to find the optimal stock portfolio:

\[ \text{subject to: } \sum_{i=1}^{n} w_i = 1, \quad w_i \geq 0, 0.01, \quad R_p > 0, \quad \sigma_p \geq 0. \]
\[ W = \alpha \cdot \sqrt{\sum \text{cov} \cdot w} \min \]
\[ (1 - \alpha) \cdot \sum R_i \cdot w_i \]
\[ \sum_{i=1}^{n} w_i = 1 \]
\[ w_i \geq 0.01 \]
\[ \sum_{i=1}^{n} R_i \cdot w_i > 0 \]
\[ \sqrt{\sum \text{cov} \cdot w} \geq 0 \] (8)

With the development of technologies and Big Data tools, Markowitz's theory can be easily applied in practice for a large set of different papers in a portfolio. Despite this, Markowitz's theory has a significant flaw—the lack of consideration for market trends. To overcome this flaw in Markowitz's theory, a modernized method of calculating the covariance matrix is applied, which is used for risk calculation—forecasting the covariance matrix of risks.

**Methodology**

**A. ARIMA model for predicting the covariance matrix**

The AutoRegressive Integrated Moving Average (ARIMA) model is a cornerstone in the field of time series forecasting, combining AutoRegressive (AR) and Moving Average (MA) components along with differencing (1) to render non-stationary data stationary. The model is formalized as ARIMA(p,d,q), where p denotes the order of the AR terms, d the degree of differencing, and q the order of the MA terms. The AR part explores the relationship between an observation and a number of lagged observations, p:

\[ AR(p) : Y_t = \alpha + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + \epsilon_t, \] (9)

where

- \( Y_t \) is the value of the time series at time t
- \( \alpha \) is a constant term that represents the intercept of the model.
- \( \phi_1, \phi_2, \phi_p \) are the coefficients of the lags of the time series up to order p.

The MA component models the error of the observation as a linear combination of error terms occurred at various times in the past, q:

\[ MA(q) : Y_t = \mu + \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \cdots + \theta_q \epsilon_{t-q}, \] (10)

where

- \( \mu \) represents the mean level of the time series, accounting for any constant trend in the data.
- \( \theta_1, \theta_2, \theta_q \) are the coefficients of the MA model, representing the weights given to past error terms.

In financial markets, the ARIMA model has been widely applied for stock price forecasting, attributing its popularity to the ability to model a wide range of time series data with or without trends and seasonality. By analyzing past price movements and volatility, ARIMA can generate forecasts that aid investors in making informed decisions. The application of ARIMA in this domain assumes that historical price movements and volatilities are indicative of future trends. This is particularly valuable in the high uncertainty environments of financial markets, where accurate forecasting can significantly impact investment strategy and risk management.

**B. GARCH model for predicting the covariance matrix**

One of the most effective approaches in this context is the use of GARCH models. These models, which are very popular, are widely used in financial analysis, especially by financial institutions. They serve as a tool for estimating the volatility of stock, bond, and market index returns. The information generated by GARCH models is important in a number of important financial aspects. Firstly, it is used to determine prices, helping to understand the likely changes in the value of various assets. In addition, this data can serve as a basis for forecasting the possible returns from various investment opportunities. In particular, they can be used to make informed decisions on asset allocation, risk hedging, and investment portfolio optimization. An important motivator for the use of GARCH models is their ability to account for heteroscedasticity. This concept indicates the irregular nature of the variability of error terms or variables in a statistical model. In cases where the variance of the error does not remain constant, the observations show a tendency to cluster rather than to be linearly distributed. Thus, the use of statistical models that assume constant variance can lead to irrelevant conclusions and predictions [5].

An ARCH(m) process is a process for which the variance at a point in time depends on the observations at the previous m points in time, and the relationship is as follows:

\[ \sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \cdots + \alpha_m y_{t-m}^2, \] (11)

where

- \( y_t \) is the value in the period t;
- \( \mu \) is the value in the period t-1;
- \( \sigma_t^2 \) is the variance in the period t;
- \( \alpha_0, \alpha_1, \ldots, \alpha_m \) are model parameters.

Before making a forecast, let's find the conditional variance in period t+1 using the formula:

\[ \sigma_{t+1}^2 = \alpha_0 + \alpha_1 y_t^2 + \beta \sigma_t^2, \] (12)

Before that, we find the conditional variance in the period t+1 using the formula:

\[ \sigma_{t+1}^2 \ (GARCH) = \alpha_0 \sum_{i=0}^{m} (\hat{\alpha}_i + \hat{\beta}_i) + (\hat{\alpha}_1 + \hat{\beta}_1)^{-1} \sigma_{t+1}^2. \] (13)

Given the conditional variances, we find the standard errors of the GARCH models for the logarithmic rates of return of each company using the formula:

\[ \text{std}_{\text{residual}} = \frac{\text{residual}}{\sigma_{t+1}^2}. \] (14)

Having obtained the residual vectors of each model, we need to find the Conditional Correlation Matrix. This matrix can be found by the formula:

\[ CCC = \frac{1}{n} \left( \text{cov}_{\text{residual}} \times \text{cov}_{\text{residual}} \right), \] (15)

where

- \( \text{cov}_{\text{residual}} \) is the covariance matrix of the model residuals for the i-th company.
cov<sub>residuals</sub><sup>T</sup> – is the transposed covariance matrix of the model residuals for the i-th company.

\( n_i \) – is the number of model residuals for the i-th company.

To predict the matrix, we use the following formula:

\[
\text{cov}_p = D \times \text{CCC},
\]

where

\( \text{cov}_p \) is the predicted covariance matrix for period \( t+1 \).

\( \text{CCC} \) is the conditional covariance matrix.

\( D \) is the predicted values of the matrix diagonal [7].

After all the key parameters of the models have been determined, we will proceed to implement the approaches of forecasting the predicted covariance matrix using two methods and optimizing stock portfolios using multivariate optimization.

**Presentation of the main research material.**

To build a portfolio, let's take the list of companies and the correspond daily stock prices of companies from Yahoo Finance using Python API.

The next step is to find the rate of return for each security.

The rate of return is calculated by the formula:

\[
\tau_i = \ln \left( \frac{p_t}{p_{t-1}} \right)
\]

Using Python software and application libraries, we will build ARIMA models for each security and make a forecast for one period. As a result, we have the following Table 1:

**Table 1**

<table>
<thead>
<tr>
<th>Ticker</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAPL</td>
<td>180.7</td>
</tr>
<tr>
<td>MSFT</td>
<td>411.3</td>
</tr>
<tr>
<td>GOOGL</td>
<td>138.5</td>
</tr>
<tr>
<td>JPM</td>
<td>186.1</td>
</tr>
<tr>
<td>V</td>
<td>282.8</td>
</tr>
<tr>
<td>MA</td>
<td>474.8</td>
</tr>
<tr>
<td>AMZN</td>
<td>176.2</td>
</tr>
<tr>
<td>DIS</td>
<td>111.0</td>
</tr>
<tr>
<td>JNJ</td>
<td>161.4</td>
</tr>
</tbody>
</table>

Based on the predicted values, we will build a covariance matrix of risks.

Given the covariance matrix, we can build an optimal portfolio. The optimal portfolio will have the following key indicators:

- Expected return of a portfolio: 0.15.
- Expected risk of a portfolio: 0.94.

For the following analysis, let’s use the GARCH model to find the covariance matrix. The matrix looks like this:
Given the covariance matrix, we can build an optimal portfolio. The optimal portfolio will have the following key indicators:

- Expected return of a portfolio: 0.15.
- Expected risk of a portfolio: 0.80.

### Table 2

<table>
<thead>
<tr>
<th>Method</th>
<th>Expected Return</th>
<th>Expected Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARIMA</td>
<td>0.15</td>
<td>0.94</td>
</tr>
<tr>
<td>GARCH</td>
<td>0.15</td>
<td>0.80</td>
</tr>
</tbody>
</table>

**Conclusions.** The main goal of building an optimal portfolio of securities is the quality of the results obtained. Given the results described above, it is worth noting that despite its complexity, the ARIMA model has advantages over the GARCH model, as it predicts higher expected risk at the same level of expected return. This quality of results is due to the consideration of market trends and a properly formulated optimization task. The portfolio is optimal, diversified, and can be used in practice.

It is worth noting that each of the models has its own characteristics, advantages, and disadvantages.

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