ЕКОНОМІКО-МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ БІЗНЕСОВИХ ПРОЦЕСІВ

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POTRFOLIO MANAGEMENT WITH TIME SERIES ANALYSIS METHODS

УПРАВЛІННЯ ПОРТФЕЛЕМ ЦІННИХ ПАПЕРІВ ЗА ДОПОМОГОЮ МЕТОДІВ АНАЛІЗУ ЧАСОВИХ РЯДІВ

The purpose of this research is to review and summarize the theoretical and methodological foundations of portfolio theory, methodological provisions for modeling an optimal portfolio using time series. To achieve the goal, the article used the modern portfolio theory of Harry Markowitz (MPT), which emphasizes the importance of diversification and quantitative assessment of risks and returns. The study also explore the adaptation of ARIMA and GARCH models for forecasting risk matrices, offering a nuanced approach to forecasting and utilizing market fluctuations to achieve optimal portfolio performance. The results of the study not only confirm the relevance of fundamental theories of portfolio management in the current environment, but also expand the toolkit for investors through the practical application of advanced time series analysis methodologies. The practical significance of the study is to provide investors with reliable strategies that will help them navigate and benefit from market dynamics, thereby maximizing investment results in the face of uncertainty inherent in financial markets.

Keywords: *portfolio management, time series, covariance matrix, dynamic modeling, investment.*

Складання оптимального портфеля акцій є важливою задачею для інвесторів, які бажають досягти максимальної доходності при певному рівні ризику. Оптимальний портфель акцій може бути складений згідно з пасивним підходом до інвестування, який передбачає реплікацію показників ринку або використання індексних фондів. Цей підхід базується на припущенні, що ринок в цілому дає прибуток з часом, тому інвестор просто намагається отримати частку цього прибутку, розподіляючи свої інвестиції між різними акціями згідно з ваговими коефіцієнтами індексу. Метою даного дослідження є огляд та узагальнення теоретико-методологічних засад портфельної теорії, методичних положень щодо моделювання оптимального портфеля з використанням часових рядів. Вивчення моделі оптимального портфеля цінних паперів набуває особливої актуальності в контексті соціальних та поведінкових змін. У той час як традиційні моделі оптимізації портфеля цінних паперів зосереджуються переважно на фінансових факторах, таких як очікувана дохідність та ризик, врахування соціальної та поведінкової динаміки дає більш повне розуміння інвестиційного ландшафту. Для досягнення поставленої мети в статті використано сучасну портфельну теорію Гаррі Марковіца (MPT), яка підкреслює важливість диверсифікації та кількісної оцінки ризиків і дохідності. Дослідження також є адаптацію моделей ARIMA та GARCH для прогнозування матриць ризиків, пропонуючи деталізований підхід до прогнозування та використання ринкових коливань для досягнення оптимальної продуктивності портфеля. Результати дослідження не тільки підтверджують актуальність фундаментальних теорій портфельного управління в сучасних умовах, але й розширюють інструментарій інвесторів завдяки практичному застосуванню передових методологій аналізу часових рядів. Практичне значення дослідження полягає в наданні інвесторам надійних стратегій, які допоможуть їм орієнтуватися і отримувати вигоду від динаміки ринку, тим самим максимізуючи результати інвестування в умовах невизначеності, притаманної фінансовим ринкам. Дана модель портфелю не залежить від суми інвестування, а отже, вона може підходити як для індивідуальних інвесторів з невеликим капіталом, так і для інвестиційних компаній з великими фінансовими ресурсами.

Ключові слова: *управління портфелем, часові ряди, коваріаційна матриця, динамічне моделювання, інвестування.*

Problem statement. A risk-averse investor wants to invest in companies that are traded on the US stock market. He wants to invest in the following companies: Apple Inc., Microsoft Corporation, Alphabet Inc., JPMorgan Chase & Co., Visa Inc., Mastercard Incorporated, Amazon.com, Inc., The Walt Disney Company, Johnson & Johnson.

The general aim to build an optimal portfolio of securities that will minimize risk, but maximize the investment results, using time series analysis approach

Analysis of recent research and publications. In the rapidly evolving financial markets, characterized by constant fluctuations and significant uncertainties, portfolio management emerges as a critical discipline for optimizing investment returns while managing risk. The crux of modern portfolio management lies in leveraging forecasted risk matrices, a concept deeply rooted in Harry Markowitz's pioneering Modern Portfolio Theory (MPT) from his 1952 dissertation "Portfolio Selection". This theory underscores the importance of diversification and quantification of risks and returns through mathematical modeling. It posits that the optimal investment portfolio can be constructed by balancing the trade-off between the expected return and the associated risk, measured in terms of variance and covariance among assets. As financial markets continue to grow in complexity, the adaptation and application of sophisticated time series analysis methods, including ARIMA and GARCH models, play a pivotal role in forecasting and managing the dynamic risks, offering investors strategic tools to navigate and capitalize on market volatility for maximizing portfolio performance.

In the current conditions, where financial markets are subject to constant changes and significant fluctuations, it is important to have an effective mechanism for managing the equity portfolio to achieve the best possible outcomes. One of the key aspects of portfolio optimization is understanding the risks associated with investing in stocks. This study explores the use of forecasted risk matrices as a tool for building effective portfolio management strategies. Such an approach allows investors not only to adapt to changes in the financial environment but also to actively use these changes to maximize profits.

The foundation for creating an optimal stock portfolio is the concept developed by American scholar Harry Markowitz. He was one of the first to fully comprehend the benefits of forming an efficient equity portfolio in his 1952 dissertation "Portfolio Selection". Modern Portfolio Theory (MPT) remains an important investment strategy, serving as an alternative to traditional stock picking. MPT provides a portfolio management tool that, when applied correctly, contributes to the formation of a diverse and profitable investment portfolio.

The main assumptions of this theory are:

– The expected return of securities is determined by the mathematical expectation of return;

– the risk of securities is determined by the standard deviation of return;

– historical data used for calculating returns and risks fully reflect future values of returns;

– the degree and nature of the correlation between securities are expressed by the coefficient of linear correlation [1; 2].

Formulating the purposes of the article. According to Markowitz's theory, the expected return of a portfolio can be determined using a formula that takes into account several factors and their interrelationships:

$$R_p = \sum_i R_i w_i, \quad (1)$$

where R_p is the return on the portfolio,

R_i – return on the asset,

w_i – share of the asset in the portfolio

The expected portfolio risk indicates how risky a portfolio is, i.e., what is the risk of losing invested funds with a certain set of securities and their weights in the portfolio. Mathematically, the expected risk of a portfolio is the standard deviation of its return. Before finding the standard deviation of the expected return, let's find its variance.

The variance measures the spread or fluctuation in the returns of the assets in a portfolio. It indicates how much the actual return of a portfolio may deviate from the expected return. The higher the variance, the greater the risk of return volatility. In the Markowitz theory, the variance is calculated using the formula:

$$\sigma_p^2 = \sum_i \sum_j w_i w_j cov\left(R_i, R_j\right) = = \sum_i \sum_j w_i w_j \sigma_{ij}, \quad (2)$$

where σ_p^2 is the variance of portfolio returns.

The expected risk of the entire portfolio is obtained by taking the square root of the variance of the return:

$$\sigma_p = \sqrt{\sigma_p^2}. \quad (3)$$

This portfolio will be built on the basis of Markowitz's portfolio theory, but with some modifications. The expected return and expected risk of the portfolio are calculated using formulas 1-3, respectively. For effective portfolio optimization, namely, optimization of both profit maximization and risk minimization, we will use multi-criteria optimization.

The first criterion is the expected return function, which will be optimized to the maximum:

$$f_1 = R_p \to max. \quad (4)$$

The second criterion is the total risk function of the portfolio, which should be optimized to a minimum:

$$f_2 = \sigma_p \to min. \quad (5)$$

Having defined the two objective functions, we can now convolve the criteria. The objective function is optimized for the minimum. Taking into account all the criteria, we have this system of constraints:

$$\begin{cases} \sum_{i=1}^{n} w_i = 1 \\ w_i \geq 0,01. \\ R_p > 0 \\ \sigma_p \geq 0 \end{cases} \quad (6)$$

The target function that will be optimized:

$$W = \frac{\alpha * \sigma_p}{\left(1-\alpha\right) * R_p} \to min. \quad (7)$$

Having defined the main criterion and constraints, we have an optimization problem to find the optimal stock portfolio:

$$W = \frac{\alpha * \sqrt{w^T * cov * w}}{(1-\alpha) * \sum_{i=1}^{n} R_i * w_i} \rightarrow min$$

$$\begin{cases} \sum_{i=1}^{n} w_i = 1 \\ w_i \geq 0,01 \\ \sum_{i=1}^{n} R_i * w_i > 0 \\ \sqrt{w^T * cov * w} \geq 0 \end{cases} . \quad (8)$$

With the development of technologies and Big Data tools, Markowitz's theory can be easily applied in practice for a large set of different papers in a portfolio. Despite this, Markowitz's theory has a significant flaw – the lack of consideration for market trends. To overcome this flaw in Markowitz's theory, a modernized method of calculating the covariance matrix is applied, which is used for risk calculation – forecasting the covariance matrix of risks.

Methodology

A. ARIMA model for predicting the covariance matrix

The AutoRegressive Integrated Moving Average (ARIMA) model is a cornerstone in the field of time series forecasting, combining AutoRegressive (AR) and Moving Average (MA) components along with differencing (I) to render non-stationary data stationary. The model is formalized as ARIMA(p,d,q), where *p* denotes the order of the AR terms, *d* the degree of differencing, and *q* the order of the MA terms. The AR part explores the relationship between an observation and a number of lagged observations, *p*:

$$AR(p): Y_t = \alpha + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \dots + \phi_p Y_{t-p} + \epsilon_t , \quad (9)$$

where

Y_t is the value of the time series at time *t*

α is a constant term that represents the intercept of the model.

ϕ_1, ϕ_2, ϕ_p are the coefficients of the lags of the time series up to order p. These coefficients measure the impact of past values of the time series on its current value.

Y_{t-1}, Y_{t-2}, Y_{t-p} are the lagged values of the time series, where Y_{t-1} is the value at time t-1, and so on, up to p periods in the past

ϵ_t is the error term at time *t*, which represents random shocks to the time series that cannot be explained by the past values. It's assumed to be white noise, meaning it has a mean of zero and a constant variance.

The MA component models the error of the observation as a linear combination of error terms occurred at various times in the past, *q*:

$$MA(q): Y_t = \mu + \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \dots + \theta_q \epsilon_{t-q} , \quad (10)$$

where

μ represents the mean level of the time series, accounting for any constant trend in the data.

θ_1, θ_2, θ_p are the coefficients of the MA model, representing the weights given to past error terms.

ϵ_{t-1}, ϵ_{t-2}, ϵ_{t-q} are the coefficients of the MA model, representing the weights given to past error terms [3; 4].

In financial markets, the ARIMA model has been widely applied for stock price forecasting, attributing its popularity to the ability to model a wide range of time series data with or without trends and seasonality. By analyzing past price movements and volatility, ARIMA can generate forecasts that aid investors in making informed decisions. The application of ARIMA in this domain assumes that historical price movements and volatilities are indicative of future trends. This is particularly valuable in the high uncertainty environments of financial markets, where accurate forecasting can significantly impact investment strategy and risk management.

B. GARCH model for predicting the covariance matrix

One of the most effective approaches in this context is the use of GARCH models. These models, which are very popular, are widely used in financial analysis, especially by financial institutions. They serve as a tool for estimating the volatility of stock, bond and market index returns. The information generated by GARCH models is important in a number of important financial aspects. Firstly, it is used to determine prices, helping to understand the likely changes in the value of various assets. In addition, this data can serve as a basis for forecasting the possible returns from various investment opportunities. In particular, they can be used to make informed decisions on asset allocation, risk hedging, and investment portfolio optimization. An important motivator for the use of GARCH models is their ability to account for heteroscedasticity. This concept indicates the irregular nature of the variability of error terms or variables in a statistical model. In cases where the variance of the error does not remain constant, the observations show a tendency to cluster rather than to be linearly distributed. Thus, the use of statistical models that assume constant variance can lead to irrelevant conclusions and predictions [5].

An ARCH(m) process is a process for which the variance at a point in time depends on the observations at the previous m points in time, and the relationship is as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \ldots + \alpha_m y_{t-m}^2 , \quad (11)$$

where y_t – value in the period *t*;

y_{t-1} – value in the period *t-1*;

σ_t^2 – variation in the period *t*;

$\alpha_0, \alpha_1, \ldots, \alpha_m$ – model parameters

Before making a forecast, let's find the conditional variance in period t+1 using the formula:

$$\sigma_{t+1}^2 = \alpha_0 + \alpha_1 y_t^2 + \beta_1 \sigma_t^2 . \quad (12)$$

After that, we find the conditional variance in the period *t+s* [6]:

$$\widehat{\sigma_{t+1}^2}(GARCH) = \widehat{\alpha_0} \sum_{i=0}^{s-2} \left(\widehat{\alpha_1} + \widehat{\beta_1} \right)^i + \left(\widehat{\alpha_1} + \widehat{\beta_1} \right)^{s-1} \widehat{\sigma_{t+1}^2} . \quad (13)$$

Given the conditional variances, we find the standard errors of the GARCH models for the logarithmic rates of return of each company using the formula:

$$std_{residual\,it} = \frac{residual_{it}}{\widehat{\sigma_{it}^2}} . \quad (14)$$

Having obtained the residual vectors of each model, we need to find the Conditional Correlation Matrix. This matrix can be found by the formula:

$$CCC = \frac{1}{n_i} \left(cov_{residuals\ i}^{\ T} \times cov_{residuals\,i} \right), \quad (15)$$

where

$cov_{residuals\,i}$ – is the covariance matrix of the model residuals for the i-th company.

$cov_{residuals\,i}^{T}$ – is the transposed covariance matrix of the model residuals for the i-th company.

n_i – is the number of model residuals for the i-th company.

To predict the matrix, we use the following formula:

$$cov_p = D \times CCC, \quad (16)$$

where

cov_p is the predicted covariance matrix for period $t+1$.

CCC is the conditional covariance matrix.

D is the predicted values of the matrix diagonal [7].

After all the key parameters of the models have been determined, we will proceed to implement the approaches of forecasting the predicted covariance matrix using two methods and optimizing stock portfolios using multivariate optimization.

Presentation of the main research material. To build a portfolio, let's take the list of companies and the correspond daily stock prices of companies from Yahoo Finance using Python API.

The next step is to find the rate of return for each security.

The rate of return is calculated by the formula:

$$r_i = \ln\left(\frac{P_i}{P_{i-1}}\right). \quad (17)$$

Using Python software and application libraries, we will build ARIMA models for each security and make a forecast for one period. As a result, we have the following Table 1:

Table 1

Predicted prices using ARIMA

Ticker	Price
AAPL	180.7
MSFT	411.3
GOOGL	138.5
JPM	186.1
V	282.8
MA	474.8
AMZN	176.2
DIS	111.0
JNJ	161.4

Based on the predicted values, we will build a covariance matrix of risks.

Given the covariance matrix, we can build an optimal portfolio. The optimal portfolio will have the following key indicators:

Expected return of a portfolio: 0.15.

Expected risk of a portfolio: 0.94.

For the following analysis, let's use the GARCH model to find the covariance matrix. The matrix looks like this:

Ticker	AAPL	AMZN	DIS	GOOGL	JNJ	JPM	MA	MSFT	V
Ticker									
AAPL	0.000156	1.098525e-04	0.000062	0.000126	1.528498e-05	0.000041	0.000063	0.000103	0.000055
AMZN	0.000110	4.147497e-04	0.000095	0.000230	3.892100e-07	0.000033	0.000079	0.000184	0.000061
DIS	0.000062	9.465201e-05	0.000304	0.000074	1.108358e-05	0.000056	0.000066	0.000059	0.000053
GOOGL	0.000126	2.298307e-04	0.000074	0.000363	1.220538e-05	0.000038	0.000067	0.000152	0.000055
JNJ	0.000015	3.892100e-07	0.000011	0.000012	1.004722e-04	0.000028	0.000021	0.000004	0.000019
JPM	0.000041	3.297089e-05	0.000056	0.000038	2.825519e-05	0.000155	0.000051	0.000018	0.000051
MA	0.000063	7.916531e-05	0.000066	0.000067	2.115527e-05	0.000051	0.000116	0.000061	0.000083
MSFT	0.000103	1.840786e-04	0.000059	0.000152	4.129702e-06	0.000018	0.000061	0.000231	0.000051
V	0.000055	6.123173e-05	0.000053	0.000055	1.892638e-05	0.000051	0.000083	0.000051	0.000093

	AAPL	AMZN	DIS	GOOGL	JNJ	JPM	MA	MSFT	V
AAPL	0.000133	0.000099	0.000055	0.000112	0.000013	0.000026	0.000057	0.000081	0.000050
AMZN	0.000099	0.000360	0.000090	0.000212	-0.000002	0.000025	0.000074	0.000146	0.000059
DIS	0.000055	0.000090	0.000304	0.000074	0.000011	0.000038	0.000065	0.000048	0.000051
GOOGL	0.000112	0.000212	0.000074	0.000343	0.000010	0.000029	0.000063	0.000123	0.000052
JNJ	0.000013	-0.000002	0.000011	0.000010	0.000098	0.000022	0.000020	0.000002	0.000018
JPM	0.000026	0.000025	0.000038	0.000029	0.000022	0.000082	0.000036	0.000014	0.000036
MA	0.000057	0.000074	0.000065	0.000063	0.000020	0.000036	0.000115	0.000054	0.000081
MSFT	0.000081	0.000146	0.000048	0.000123	0.000002	0.000014	0.000054	0.000171	0.000045
V	0.000050	0.000059	0.000051	0.000052	0.000018	0.000036	0.000081	0.000045	0.000091

Given the covariance matrix, we can build an optimal portfolio. The optimal portfolio will have the following key indicators:

Expected return of a portfolio: 0.15.

Expected risk of a portfolio: 0.80.

Table 2

Comprasion of results

Method	Expected Return	Expected Risk
ARIMA	0.15	0.94
GARCH	0.15	0.80

Conclusions. The main goal of building an optimal portfolio of securities is the quality of the results obtained. Given the results described above, it is worth noting that despite its complexity, the ARIMA model has advantages over the GARCH model, as it predicts higher expected risk at the same level of expected return. This quality of results is due to the consideration of market trends and a properly formulated optimization task. The portfolio is optimal, diversified, and can be used in practice.

It is worth noting that each of the models has its own characteristics, advantages, and disadvantages.

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